Hot Electrons

A Myth or Reality

by

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Our Motivation and Economics

Adam Smith, “An Enquiry into Nature and Causes of the Wealth of Nations” (1776)

The wealth is created by laisse-faire economy and free trade

John Maynard Keynes, “The General Theory of Employment, Interest, and Money” (1936)

The wealth is created by careful government planning and government stimulation of economy

1990’s and Beyond

The wealth is created by innovations and inventions

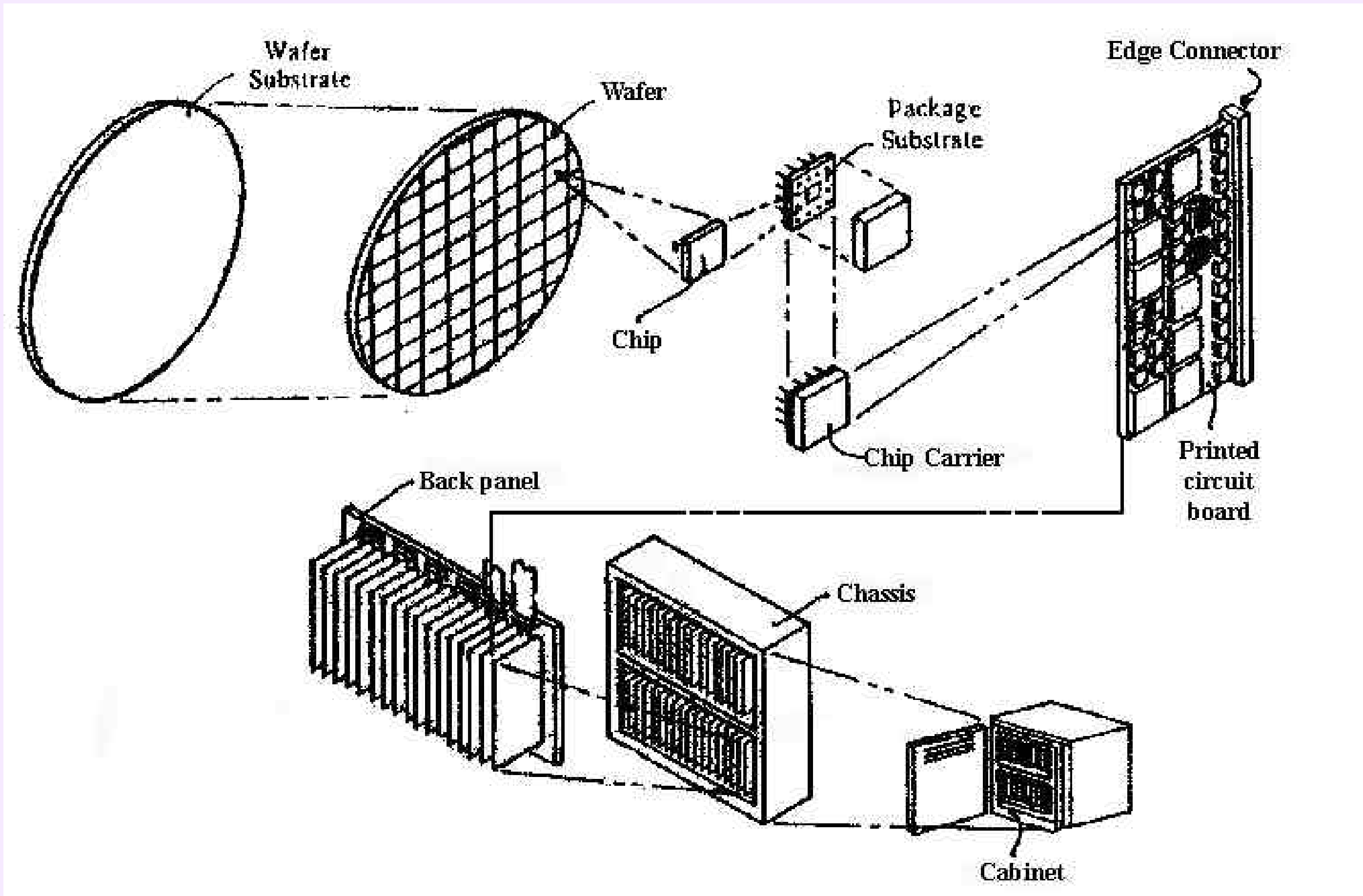
20th Century Paradigm

- Formulate a Hypothesis or Theory
- Accumulate data
- Do Extensive experimentation and Check
- Publish if newsworthy

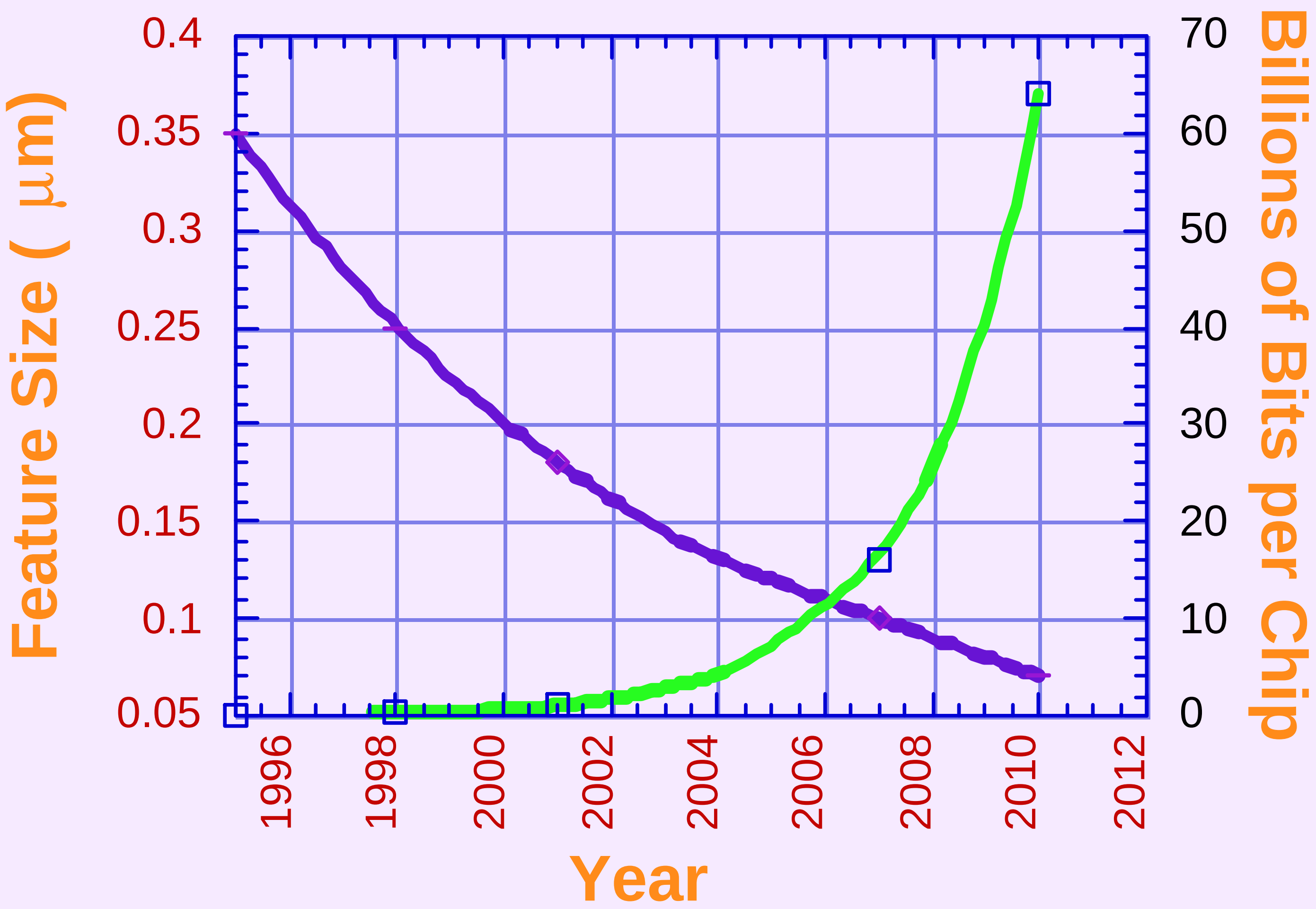
21st Century Paradigm

- Formulate a Hypothesis or theory or design
- Make a prototype structure
- Patent it
- Raise 17 million dollars and start an IPO
- Sue your competitor for stealing your idea

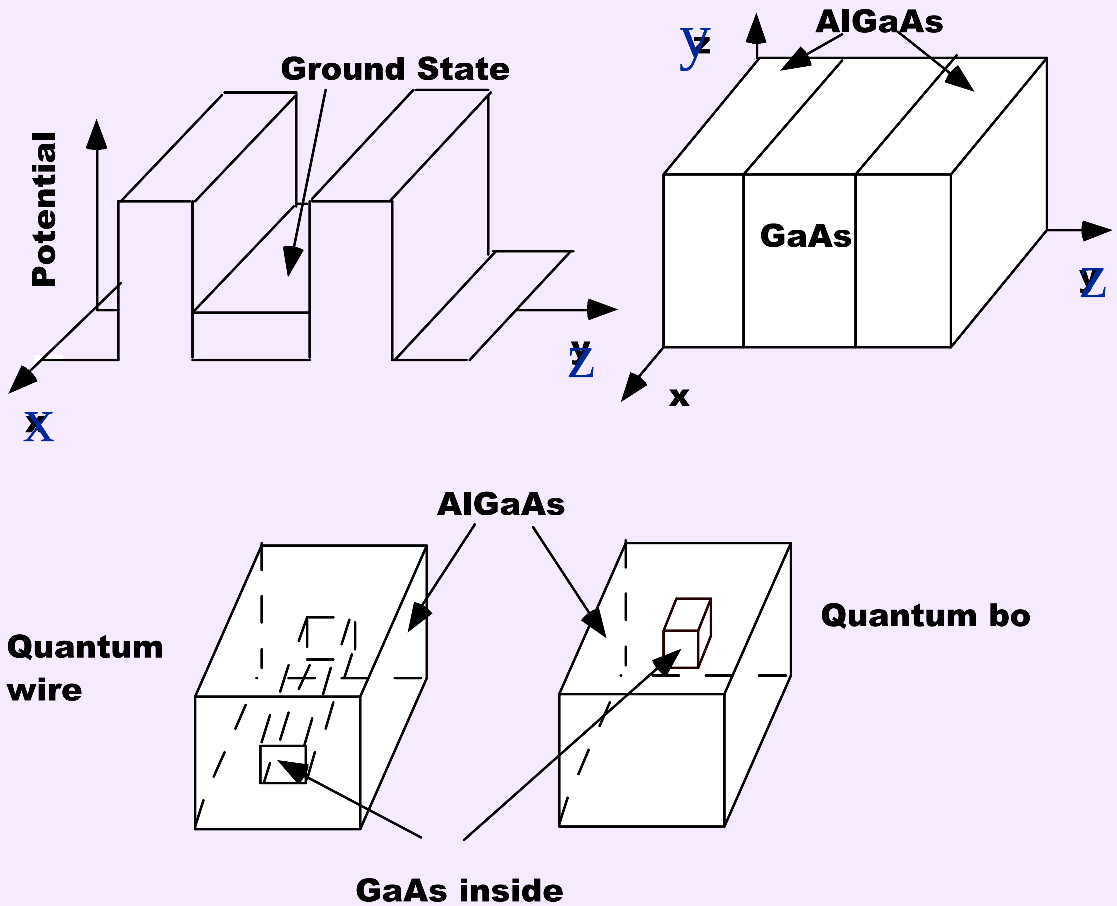
Micro-Journey: From Wafer to Cabinet



Downsizing on a Chip



Quantum Nanostructures



Quantum Nanostructures

$$\varepsilon_{k_x k_y k_z} = \varepsilon_c + \frac{\hbar^2\left(k_x^2 + k_y^2 + k_z^2\right)}{2m^*} \qquad (3D\ \ Bulk)$$

$$\varepsilon_{nk_y k_z} = \varepsilon_c + \varepsilon_n + \frac{\hbar^2\left(k_y^2 + k_z^2\right)}{2m^*} \qquad (Q2D)$$

$$\varepsilon_{nmk_z} = \varepsilon_c + \varepsilon_{nm} + \frac{\hbar^2 k_z^2}{2m^*} \qquad (Q1D)$$

$$\varepsilon_{nmk_z} = \varepsilon_c + \varepsilon_{nm\ell} \qquad (Q0D\ \ or\ Quantum\ Dot)$$

High-Field Effects

- Electric Field in a Macro-Device

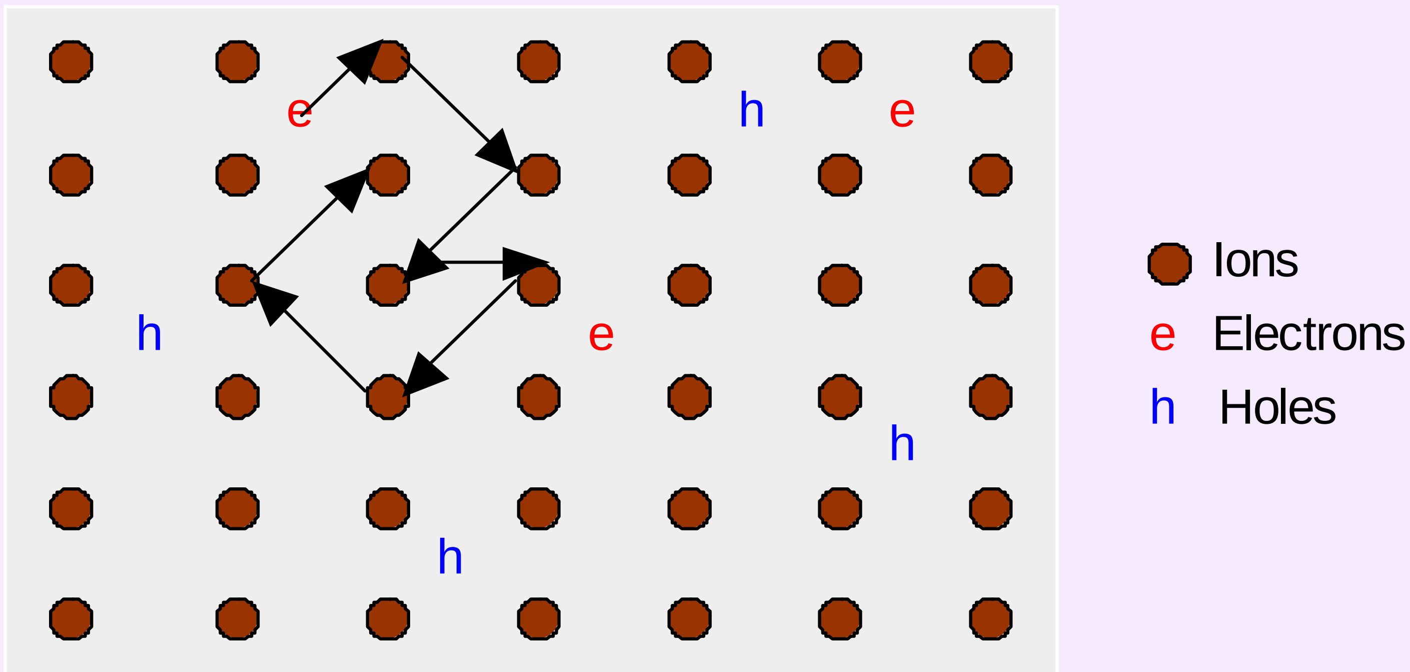
$$\varepsilon = \frac{V}{L} = \frac{5\ V}{1\ cm} = 5\frac{V}{cm}$$

- Electric Field in a Micro-Device of Today

$$\varepsilon = \frac{V}{L} = \frac{5\ V}{1\ \mu m} = 5\frac{V}{\mu m} = 50\frac{kV}{cm}$$

- Field Broadening can become larger than collision broadening: $q\varepsilon\lambda_D >> \frac{\hbar}{\tau}$

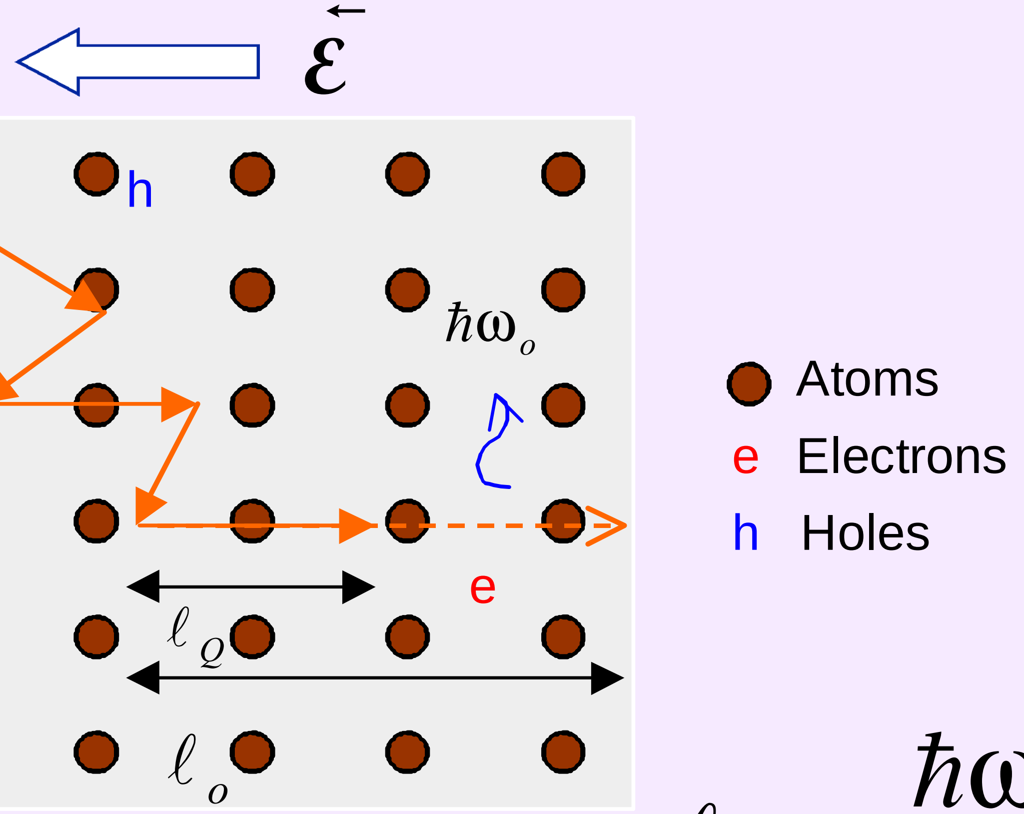
Random Thermal Motion



$$\langle\langle \vec{v}_{th} \rangle\rangle = 0$$

$$v_{th} = \sqrt{\frac{3k_B T}{m^*}} \approx 10^5\, m/s$$

Quantum Emission



$$q\mathcal{E}\ell_Q = \hbar\omega_o$$

$$\ell_Q = \frac{\hbar\omega_o}{q\mathcal{E}}$$

Modeling Transport

$$\frac{dv}{dt} = -\frac{q\varepsilon}{m^*} - \frac{v - \langle v_{th} \rangle}{\tau_c}$$

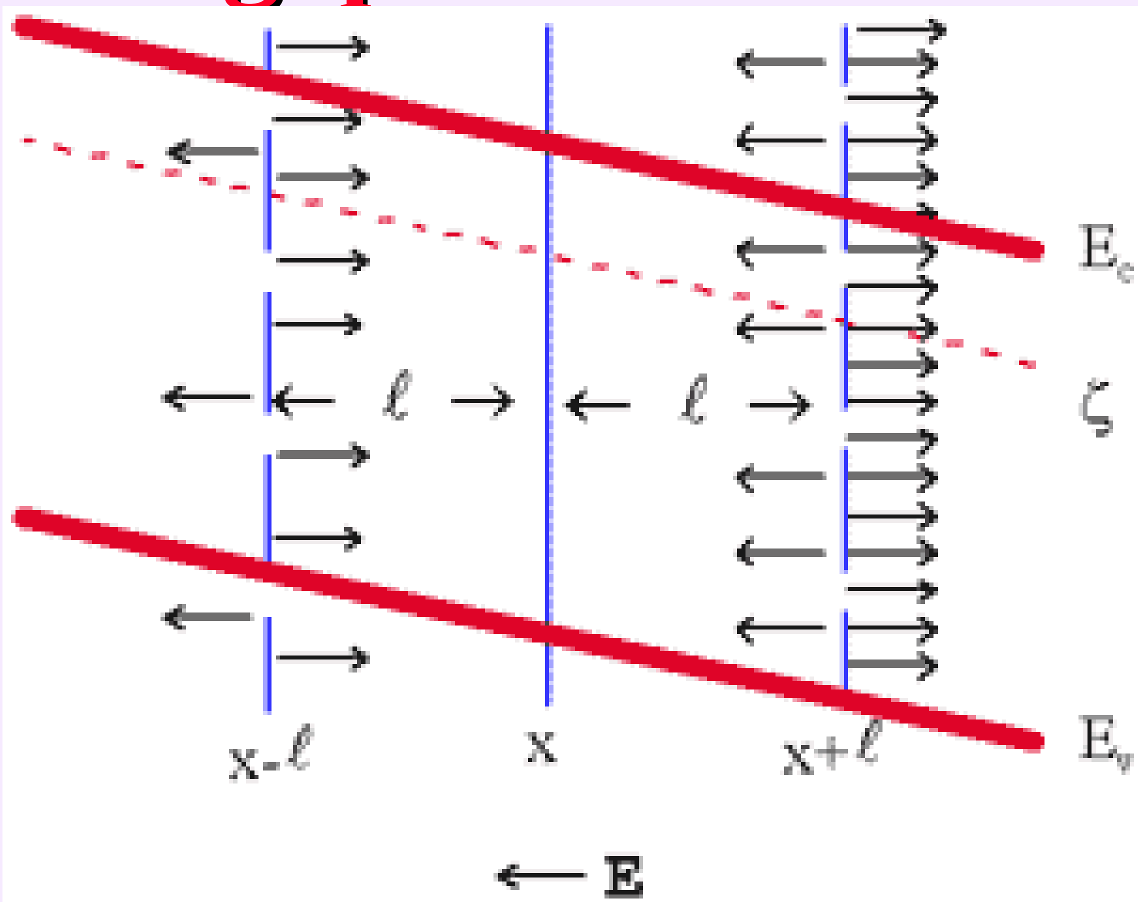
$\langle v_{th} \rangle$ =0

Transient Response: $v = -\frac{q\tau_c}{m^*}\varepsilon\left(1 - e^{-\frac{t}{\tau_c}}\right) = -\mu_o\varepsilon\left(1 - e^{-\frac{t}{\tau_c}}\right)$

Effective Collision time: $\tau = \tau_c\left(1 - e^{-\frac{\tau_Q}{\tau_c}}\right)$ $\tau_Q = \frac{\hbar\omega_o}{q\varepsilon v_{th}}$

Effective collision length $\ell = \ell_o\left(1 - e^{-\frac{\ell_Q}{\ell o}}\right)$ $\ell_Q = \frac{\hbar\omega_o}{q\varepsilon}$

1-D Random Walk in a Bandgap semiconductor



Randomness to Streamlining

Velocity Vectors in Equilibrium (Randomness):

$$\vec{v}_d = \langle \vec{v}_{th} \rangle = 0$$

Velocity Vectors in a Very High Field (Streamlined):

$$\vec{v}_d = \langle \vec{v}_{th} \rangle = -v_{th}\hat{\varepsilon}$$

Saturation Velocity

$$\langle v \rangle_{3D} = \frac{2}{\sqrt{\pi}} v_{th} \frac{\mathbf{F}_1(\eta)}{\mathbf{F}_{1/2}(\eta)}, \quad \text{where } \eta = \frac{\zeta}{k_B T}$$ Bulk

$$\langle v \rangle_{2D} = \frac{\sqrt{\pi}}{2} v_{th} \frac{\mathbf{F}_{1/2}(\eta)}{\ln(1+e^{\eta})}, \text{ where } \quad \eta = \frac{\zeta - \varepsilon_o}{k_B T}$$ Q2D

$$\langle v \rangle_{1D} = \frac{v_{th}}{\sqrt{\pi}} \frac{\ln(1+e^{\eta})}{\mathbf{F}_{-1/2}(\eta)}, \quad \text{where } \quad \eta = \frac{\zeta - \varepsilon_{oz} - \varepsilon_{oy}}{k_B T}$$ Q1D

Fermi Integral:

$$F_j(\eta) = \frac{1}{\Gamma(j+1)} \int_0^{\infty} \frac{x^j}{1+e^{x-\eta}}$$

Nondegenerate Limit Saturation Velocity with Quantum Emission

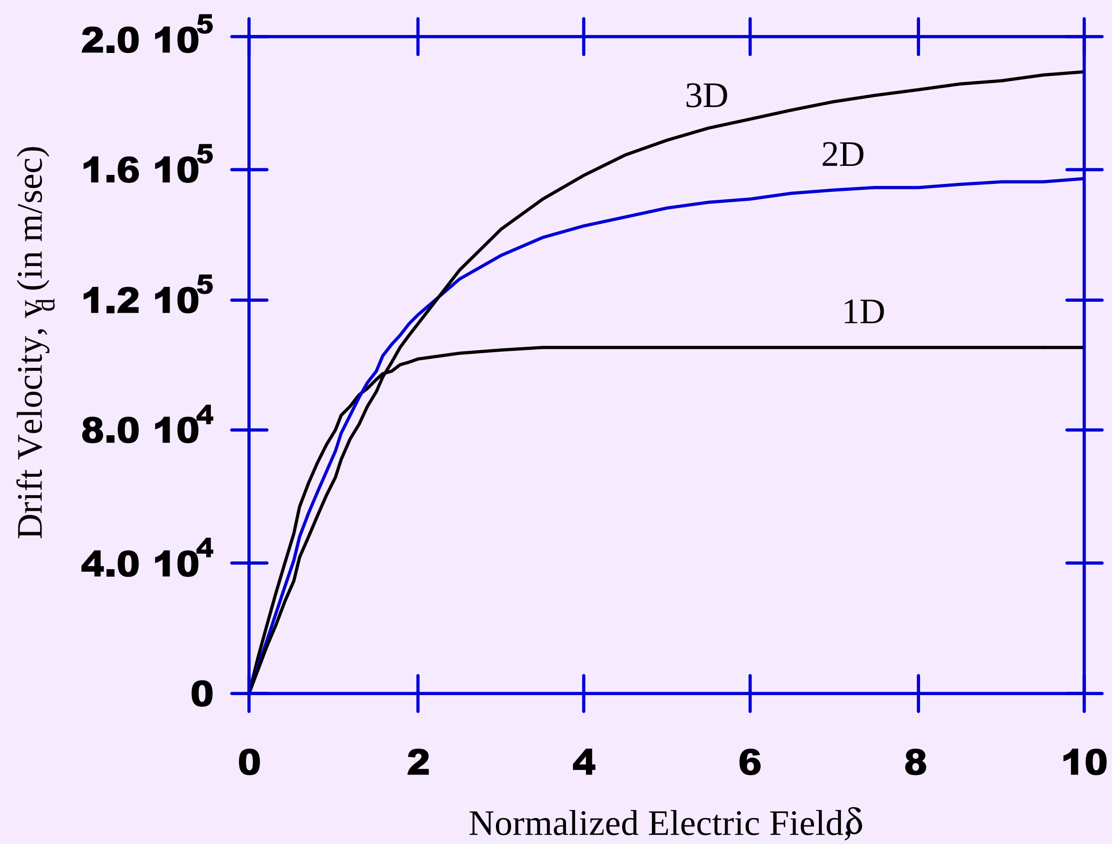
$$v_{sat3} = v_{th3}\left[\coth(\delta_Q) - \frac{1}{\delta_Q}\right] \quad 3D\ \ Bulk$$

$$v_{sat2} = v_{th2}\frac{I_1(\delta_Q)}{I_o(\delta_Q)} \qquad Q2D$$

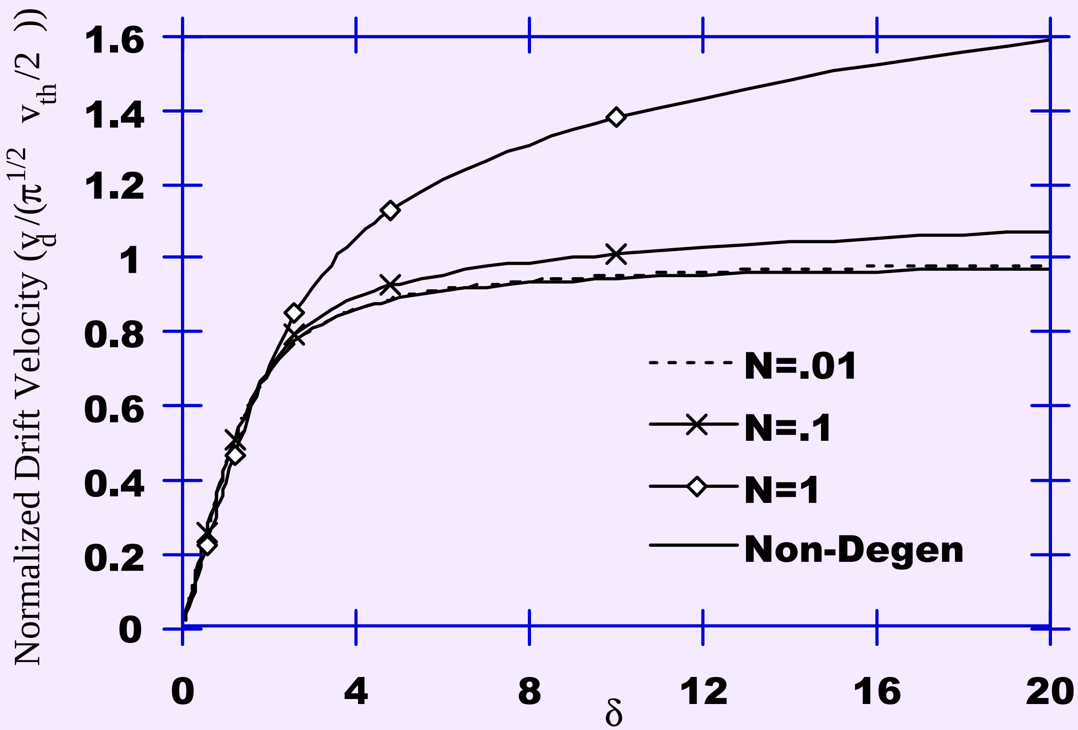
$$v_{sat1} = v_{th1}\tanh(\delta_Q) \quad Q1D$$

$$\delta_Q = \frac{\hbar\omega_o}{k_B T} \qquad Quantum\ \ Emission\ Parameter$$

Velocity-Field Characterisitcs



Effect of Degeneracy (2-D)



$$N = n_s \lambda_D^2$$

$$\lambda_D = \frac{h}{\sqrt{2m^* k_B T}}$$

Empirical Relation

Drift Velocity:

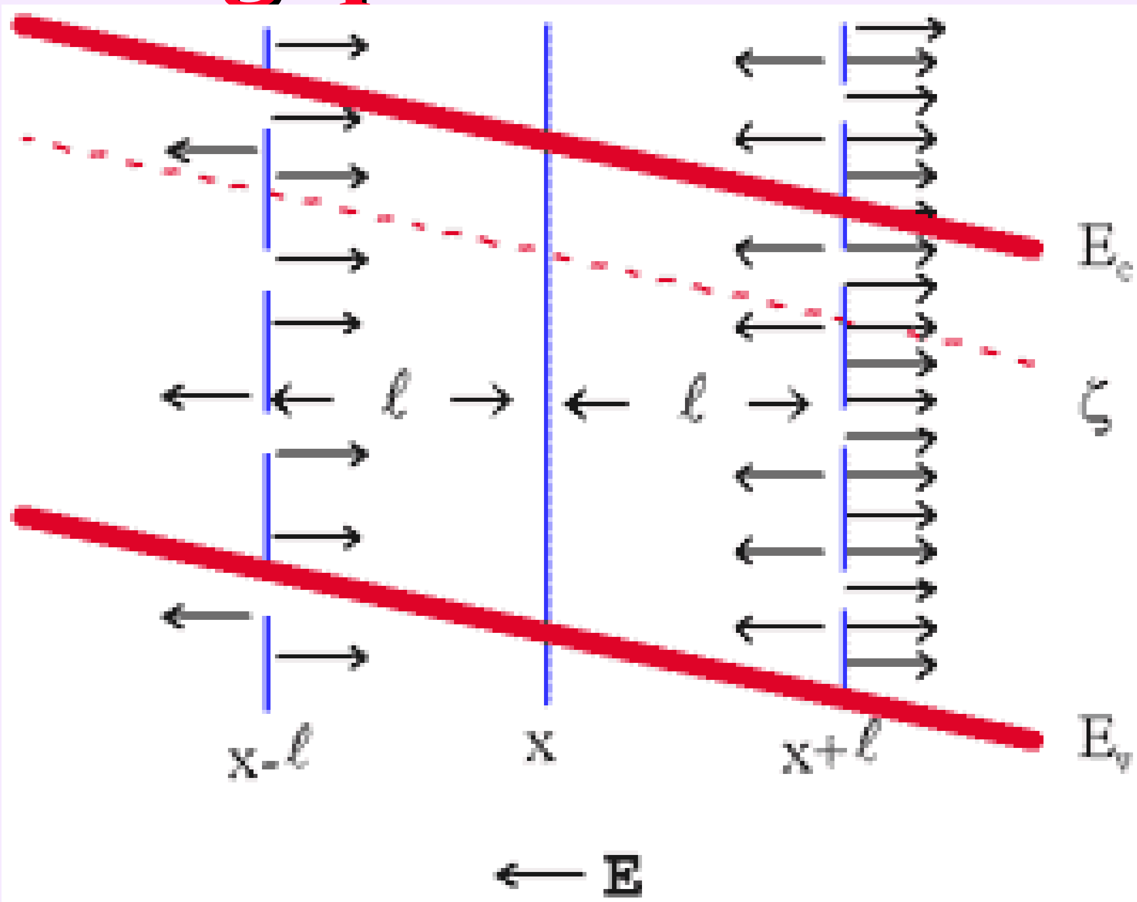
$$v_d(\varepsilon) = \frac{\mu_o \varepsilon}{1 + \frac{\varepsilon}{\varepsilon_c}} \qquad \varepsilon_c = \frac{v_{sat}}{\mu_o}$$

I-V Characteristics of a Micro-resistor (L=1 μm):

$$I = \frac{V}{R_o} \frac{1}{1 + \frac{V}{V_c}} \qquad V_c = \varepsilon_c L = \frac{v_{sat}}{\mu_o} L$$

$$V_c = \frac{10^5 m/s}{0.1 \frac{m^2}{V-s}} 10^{-6} m = 1V$$

1-D Random Walk in a Bandgap semiconductor



Modeling the Distribution

$$f(\varepsilon_\alpha, \vec{\mathcal{E}}) = \frac{1}{e^{\frac{\varepsilon_\alpha - \zeta + q\vec{\mathcal{E}}\cdot\vec{\ell}}{k_B T}} + 1}$$

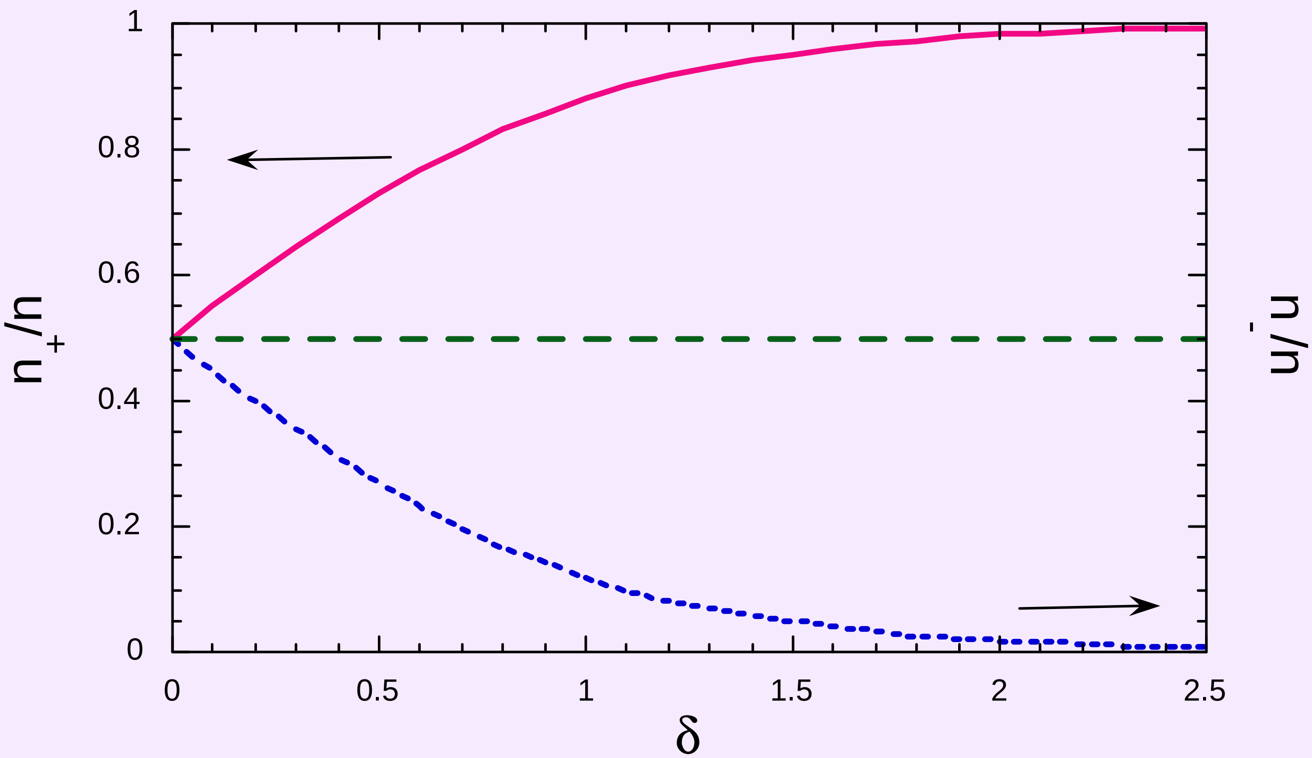
$$\frac{n_\pm(x)}{n(x)} = \frac{e^{\pm\delta}}{e^{+\delta} + e^{-\delta}} = \frac{e^{\pm\delta}}{2\cosh(\delta)}$$

$$\delta = \delta_o \left(1 - e^{-\frac{\delta_Q}{\delta_o}} \right)$$

$$\delta_o = \frac{q\mathcal{E}\ell_o}{k_B T} = \frac{\mathcal{E}}{\mathcal{E}_c} = \frac{V}{V_c}$$

$$\delta_Q = \frac{\hbar\omega_o}{k_B T}$$

Streamlining the Randomness



Drift-Diffusion

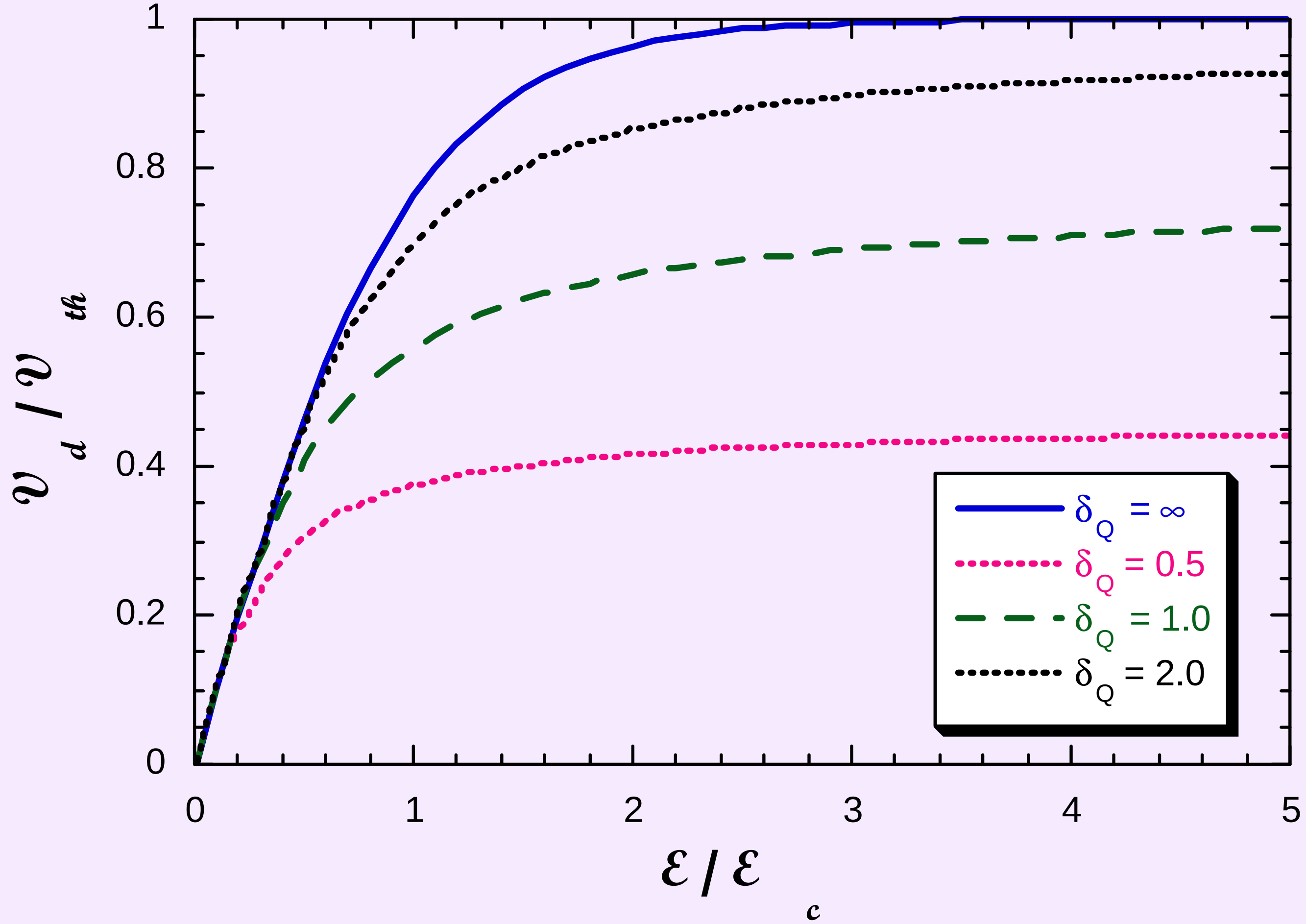
$$J(x) = n(x)\, q\, v_{th} \tanh(\delta) + q\, v_{th}\, \ell \frac{dn}{dx}$$

$$v_d = v_{th} \tanh(\delta)$$

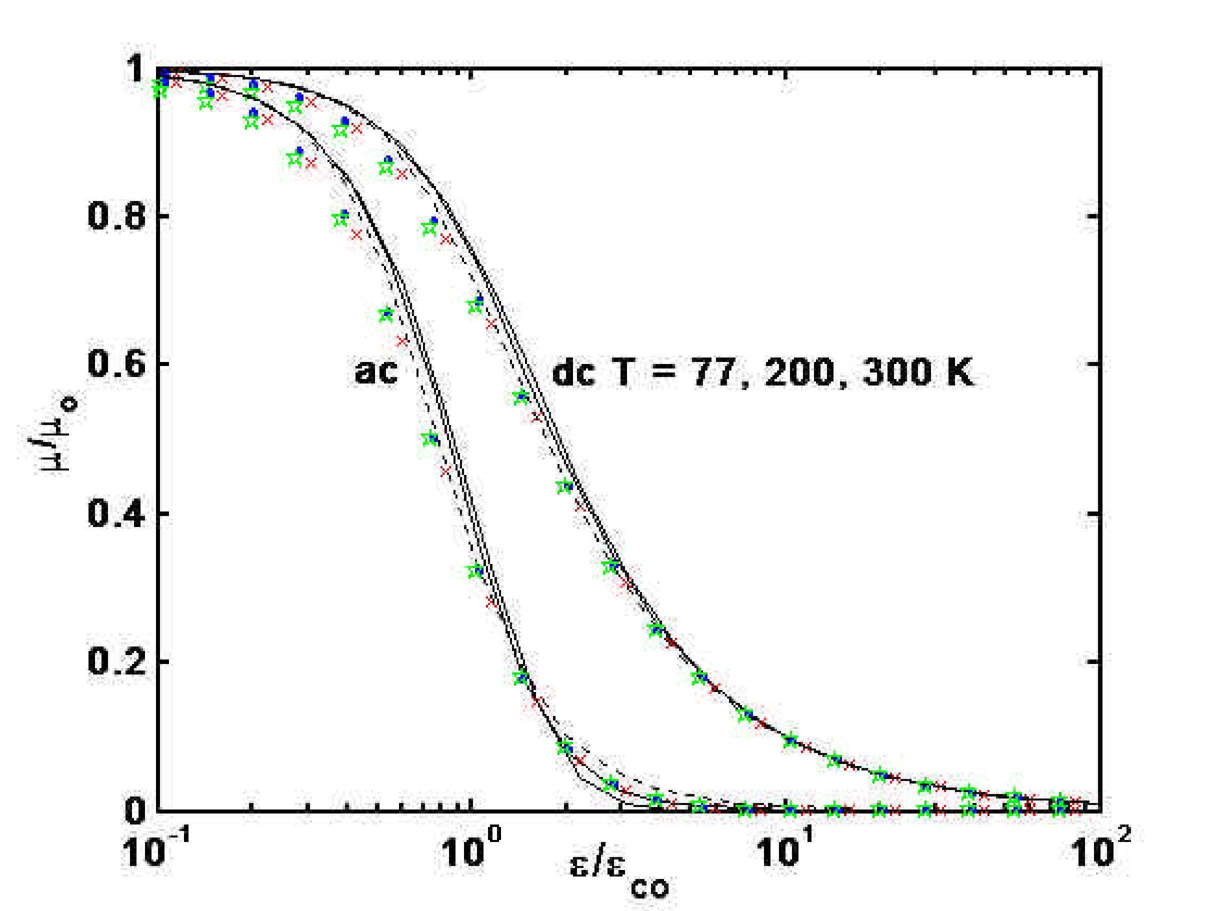
$$D_n = v_{th}\ell = \mu_{no} V_t \frac{\delta}{\delta_o}$$

$$\mu_{no} = \frac{q\, \ell_o}{m_n^* v_{th}}$$

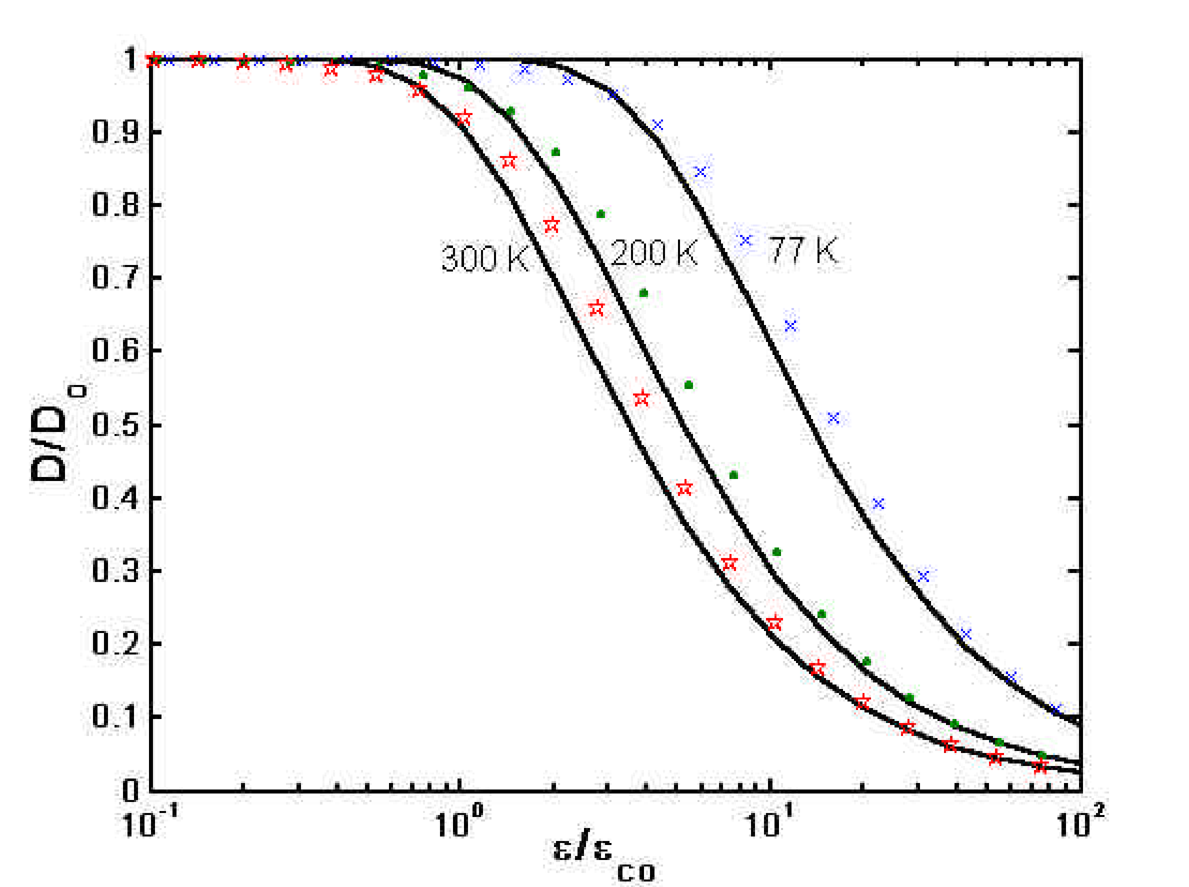
Velocity-Field Characteristics



Mobility Degradation



Diffusion Coefficient Degradation



Hot Electron Temperature

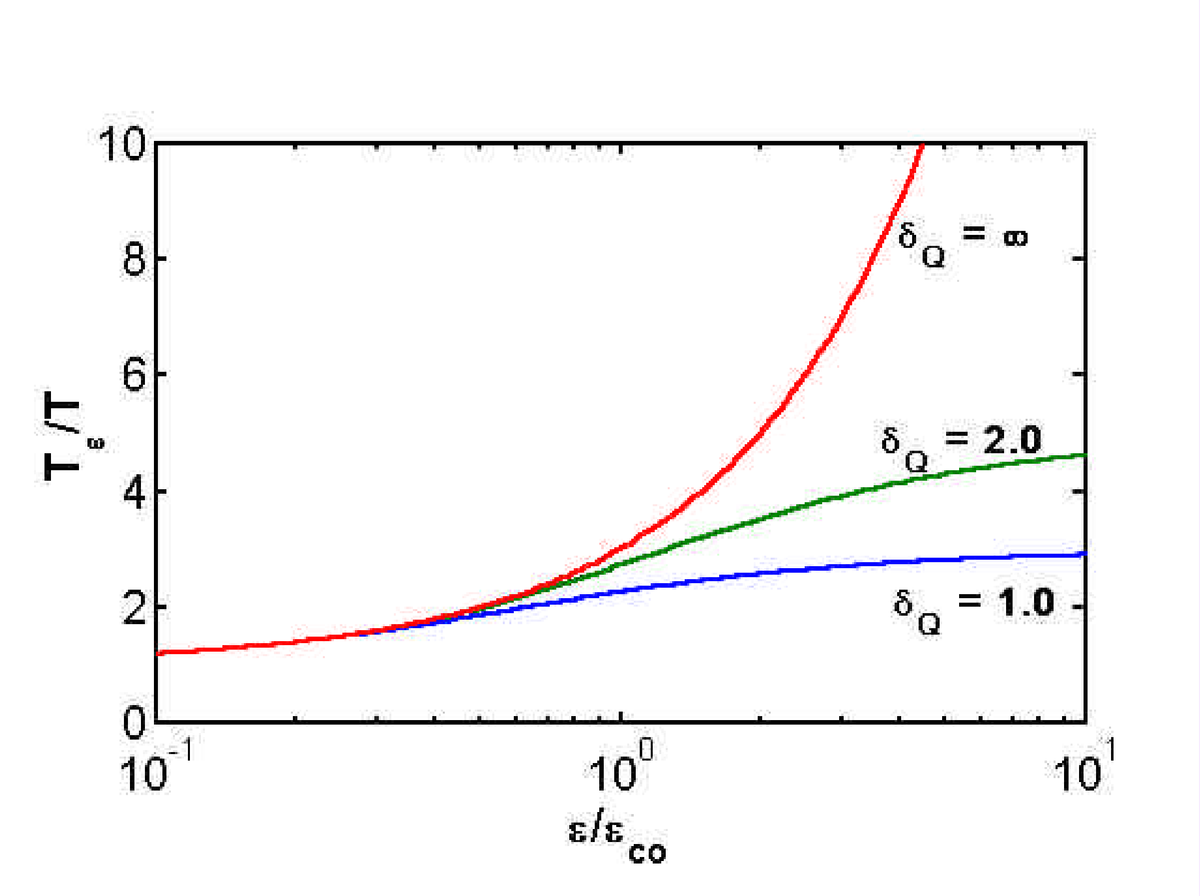
- Temperature —a measure of entropy, randomness, or chaos in a stochastic process.
- Does an electric field enhance randomness or streamline electrons?
- Does confining electric field at the gate of a MOSFET makes electrons hot?
- What happens when height and width of the barriers change at the interface?

Hot-Electron Energy Ansatz

$$\frac{1}{2}k_B T_\varepsilon = \frac{1}{2}k_B T + q\varepsilon\ell$$

$$\frac{T_\varepsilon}{T} = 1 + 2\frac{q\varepsilon\ell}{k_B T} = 1 + 2\delta$$

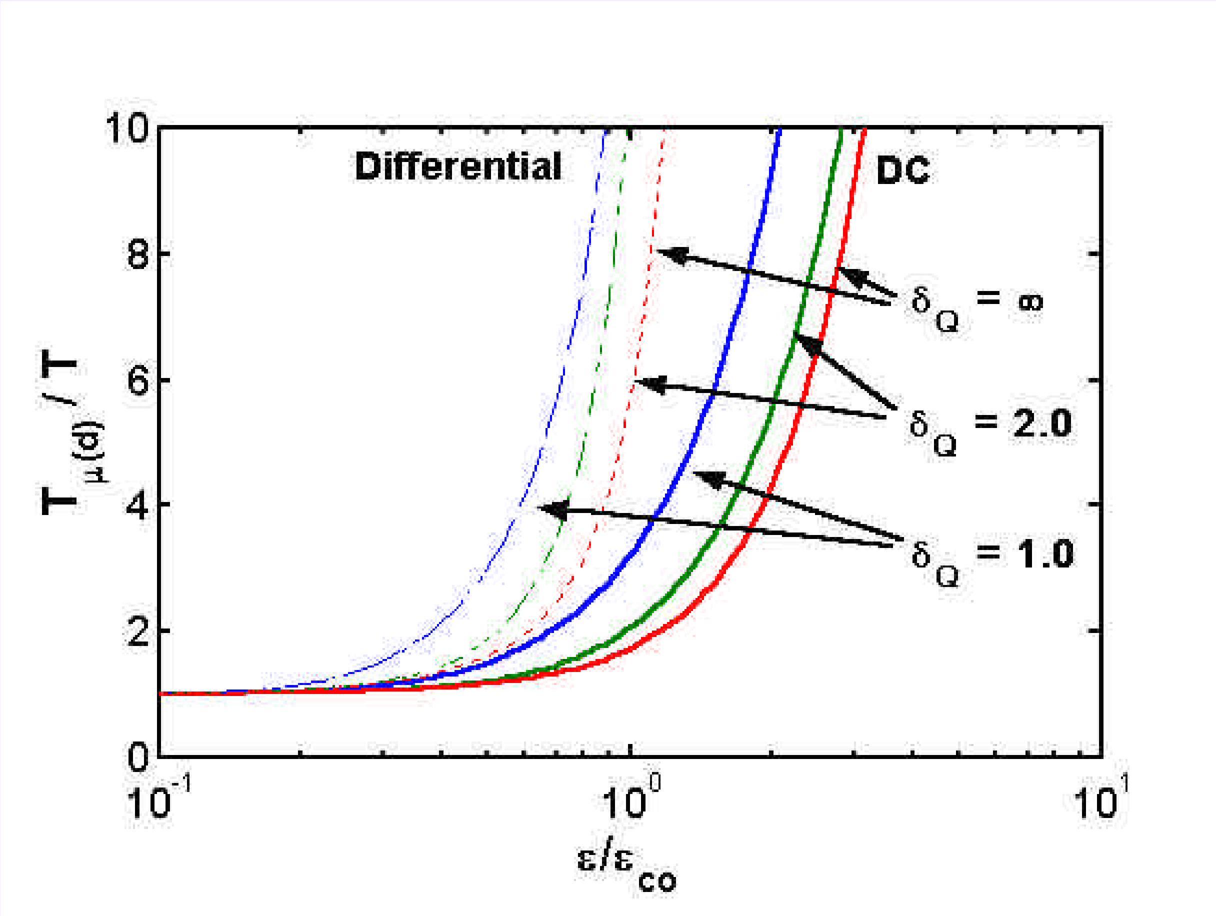
Energy Temperature



Hot-Electron Mobility Ansatz

$$\frac{T_{\mu(d)}}{T} = \left(\frac{\mu_o}{\mu_{(d)}} \right)^2$$

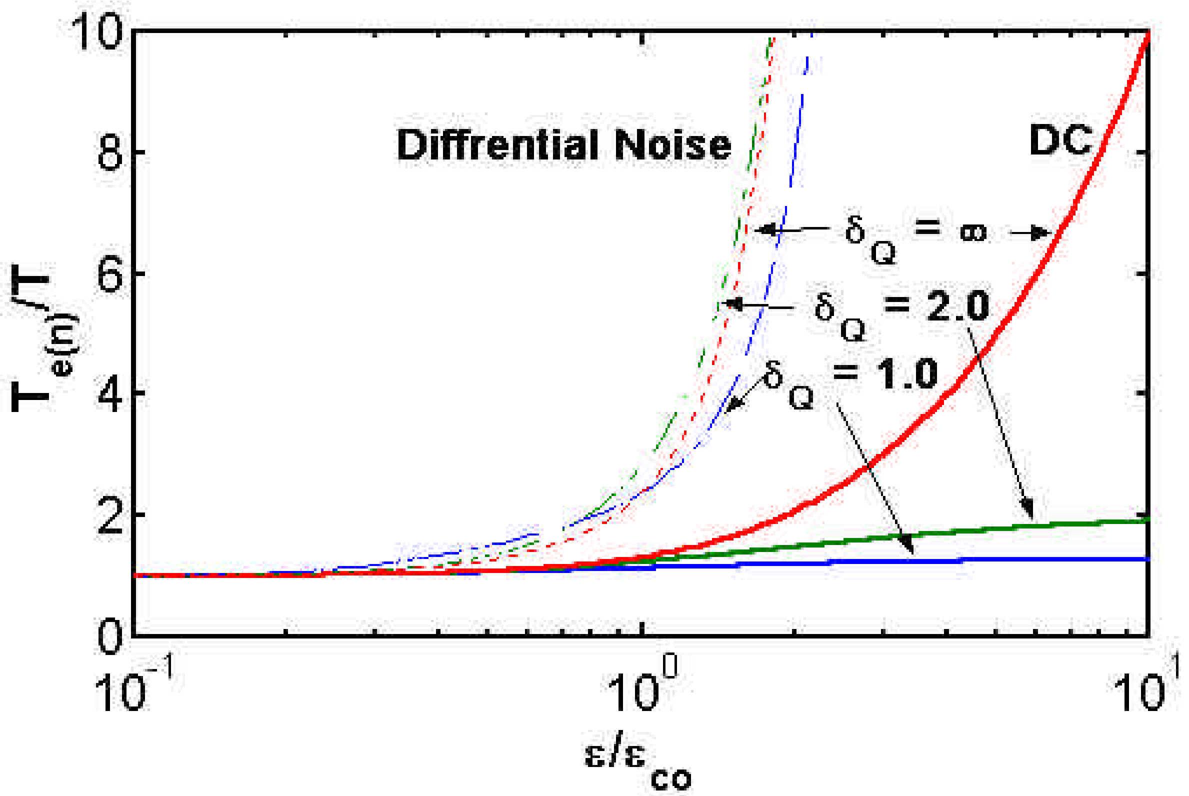
Mobility Temperature



Hot-Electron Einstein Ratio

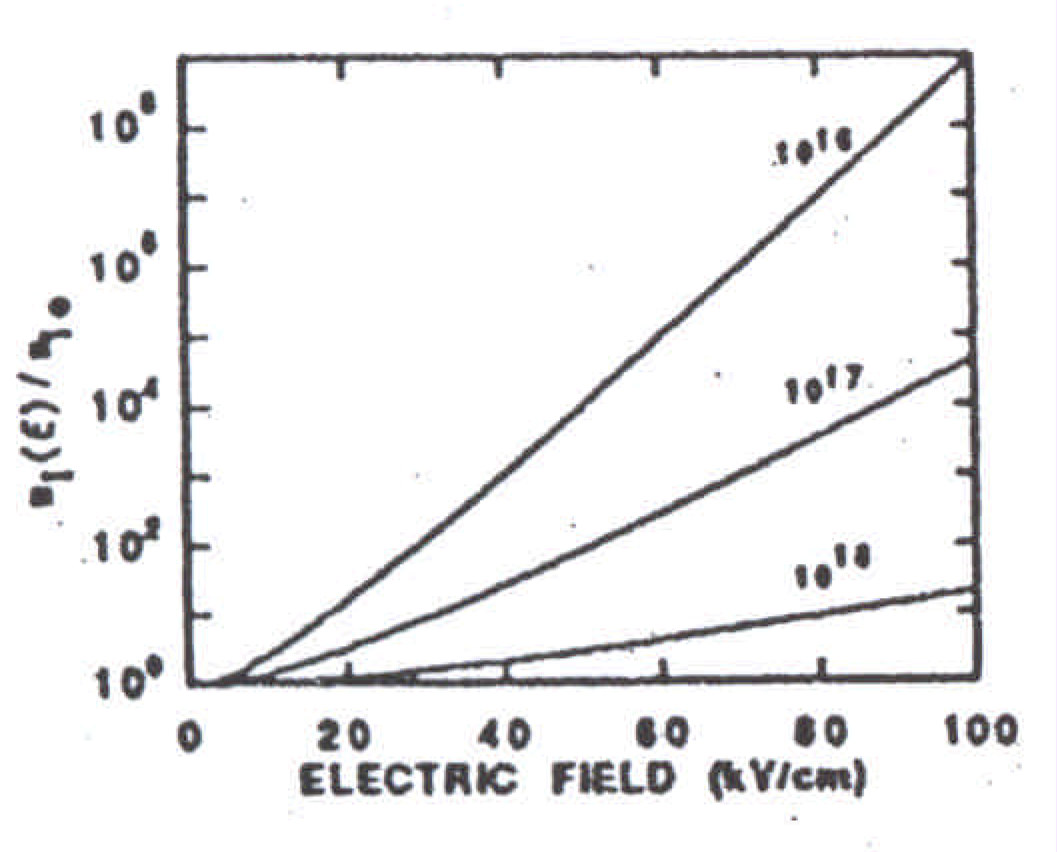
$$\frac{D(\varepsilon)}{\mu(\varepsilon)} = \frac{k_B T(\varepsilon)}{q}$$

Einstein Ratio Temperature



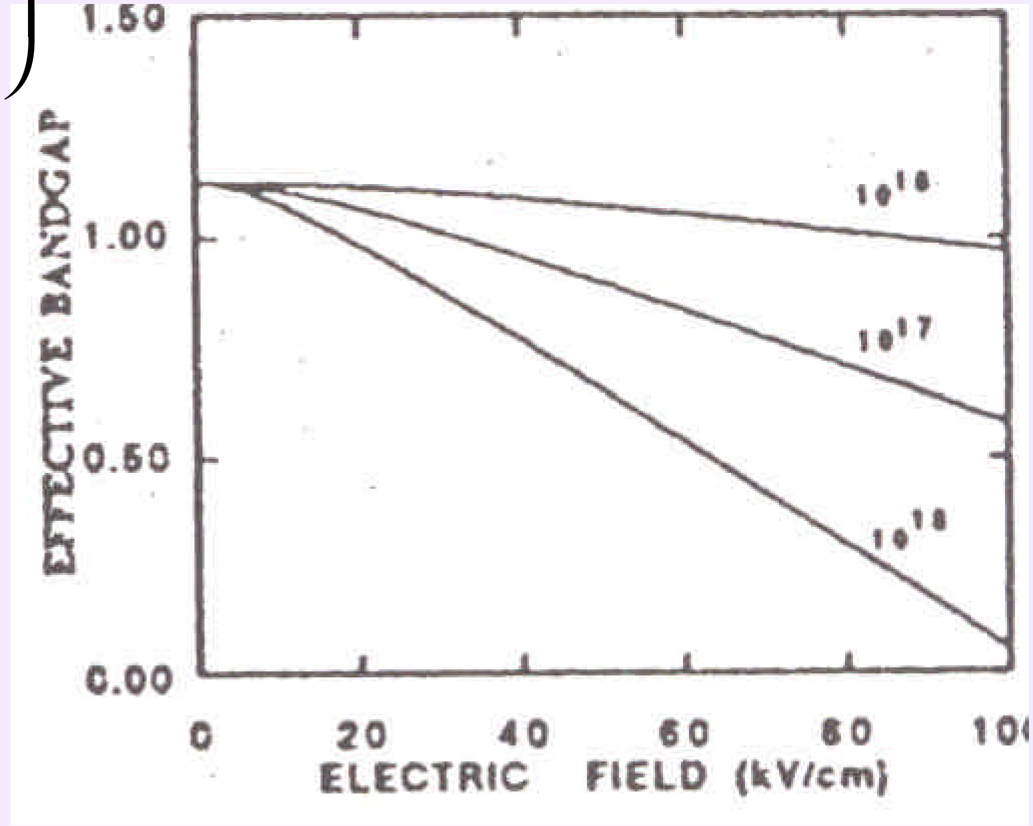
Intrinsic Carrier Multiplication

$$\frac{n_i(\varepsilon)}{n_{io}} = \left[\frac{\sinh(\delta_e)\sinh(\delta_h)}{\delta_e \delta_h}\right]^{1/2}$$



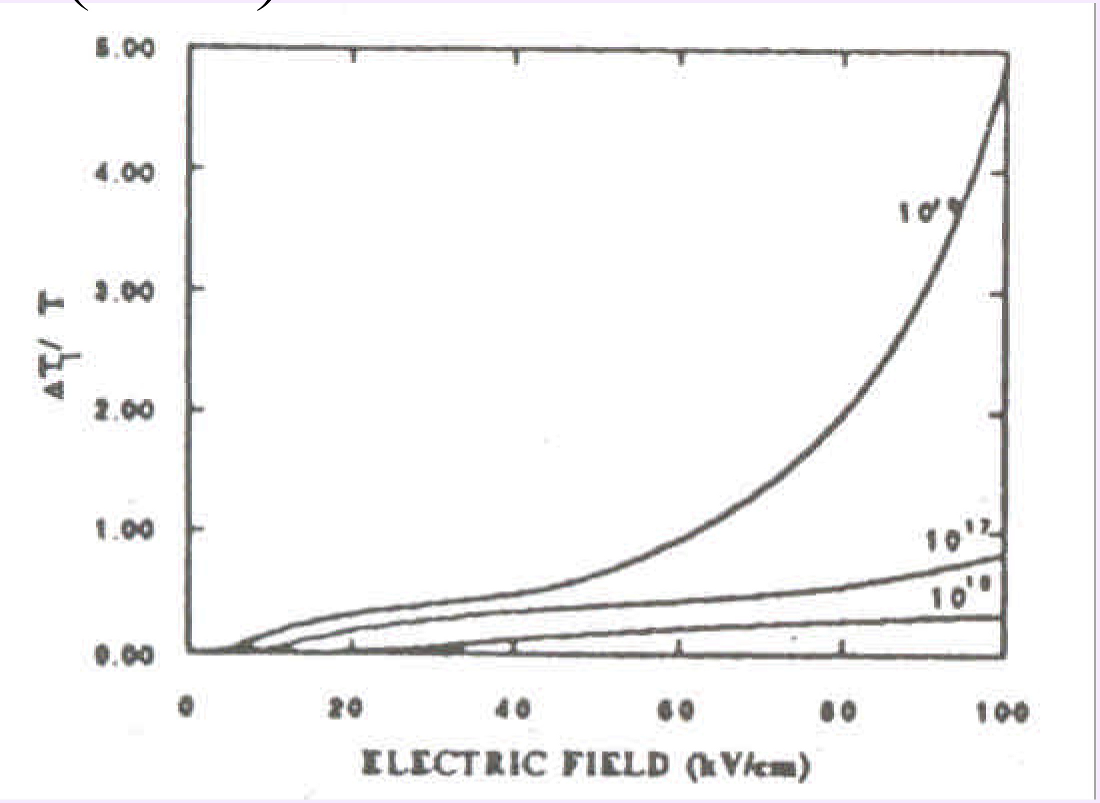
Bandgap Narrowing

$$\varepsilon_{geff} = \varepsilon_g - 2k_B T \ln\left(\frac{n_i(\varepsilon)}{n_{io}}\right)$$



Intrinsic Temperature

$$\frac{n_i(\varepsilon)}{n_{io}} = \left(\frac{T_i}{T}\right)^{3/2} \exp\left[-\frac{E_g}{2k_B}\right]\left(\frac{1}{T_i}-\frac{1}{T}\right)$$



Gate Heating of Electrons?

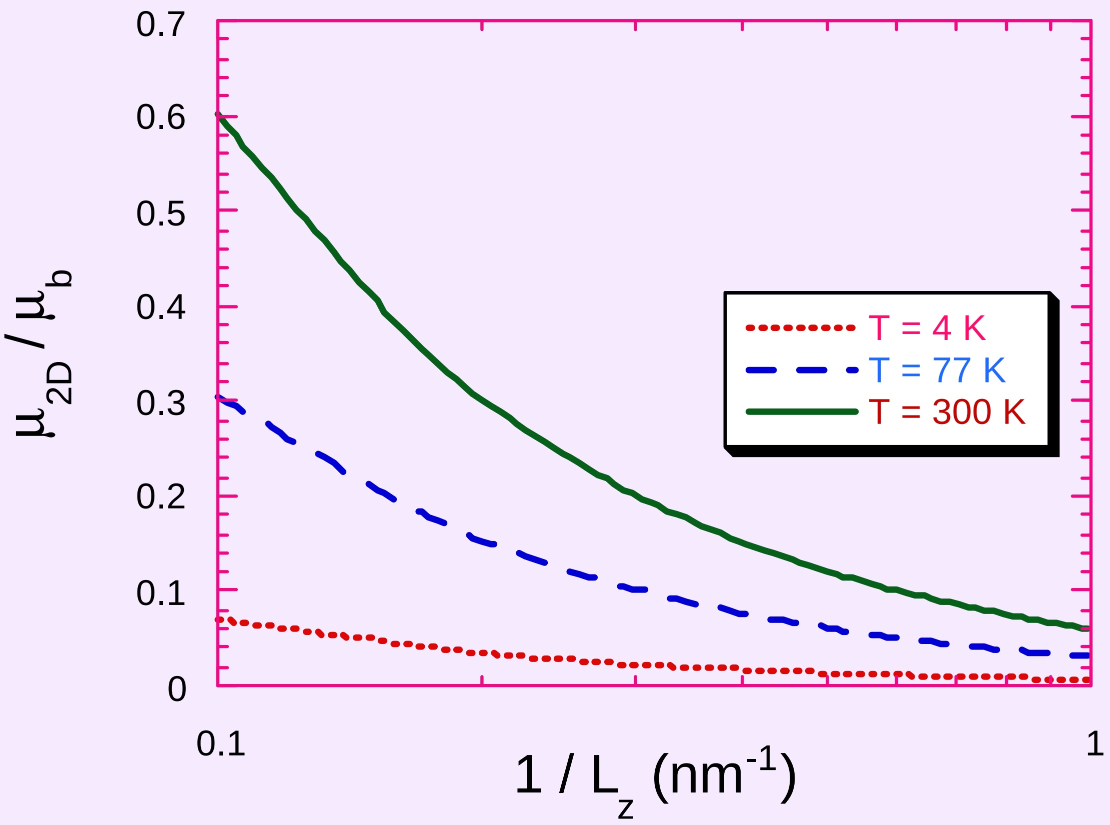
- Quantum confinement of 2D gas in an FET
- Mobility degradation
- $L_z < \lambda_D$

$$\left.\frac{\mu_{RQW}}{\mu_b}\right|_{AP} = \frac{L_z \pi^{1/2}}{\lambda_D}$$

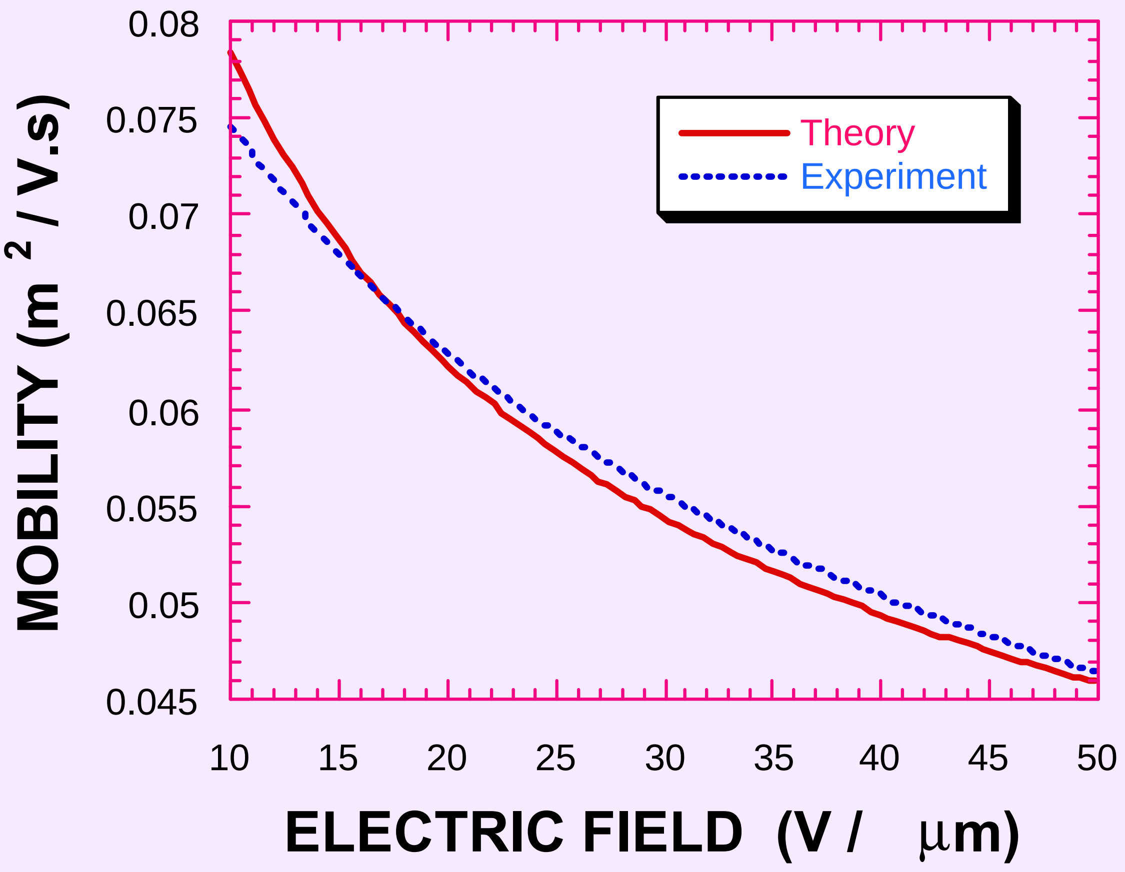
- L_z decreases with the gate electric field for a MOSFET:

$$L_z \alpha\, \varepsilon^{-1/3}$$

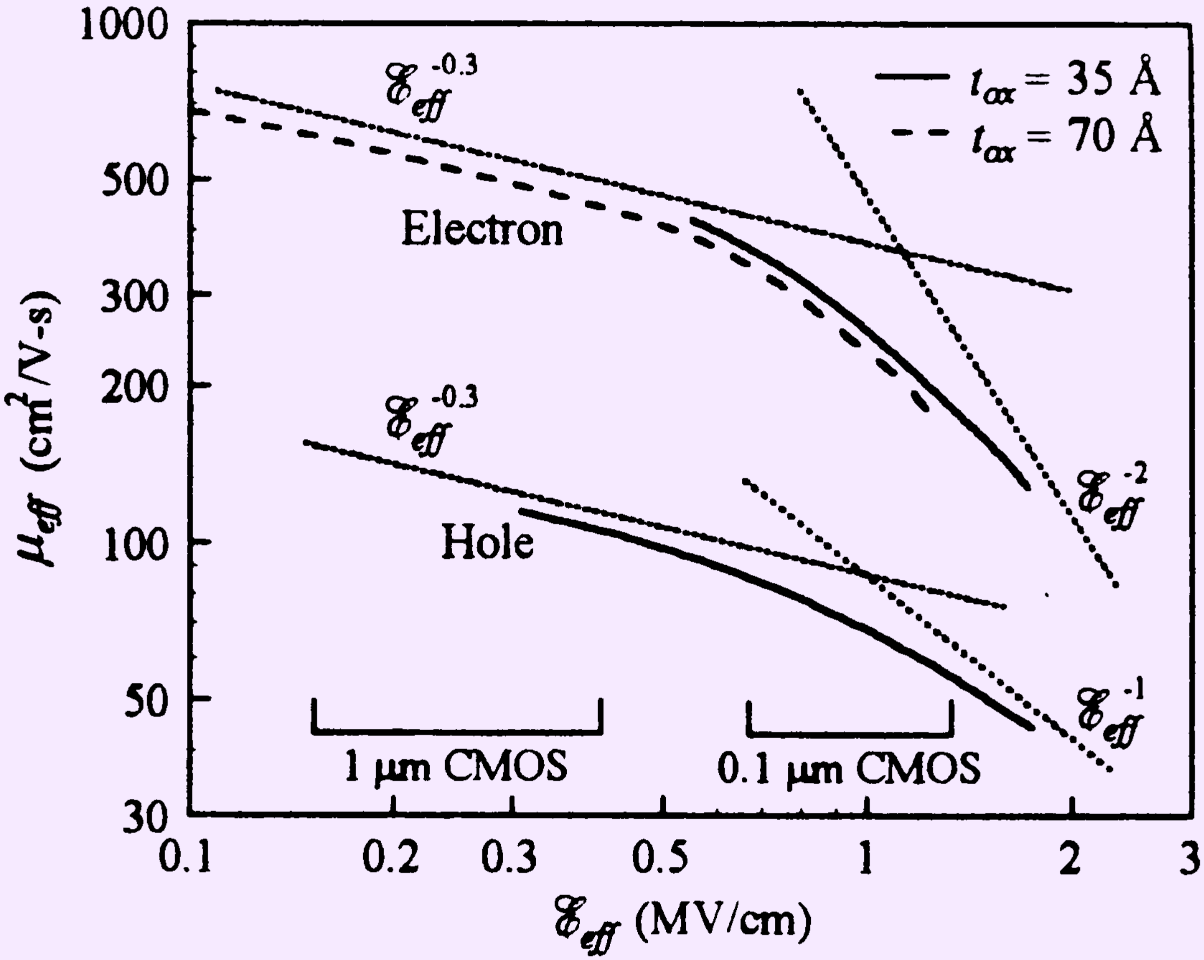
Mobility Degradation Versus Quantum Confinement



Mobility Degradation in TQW

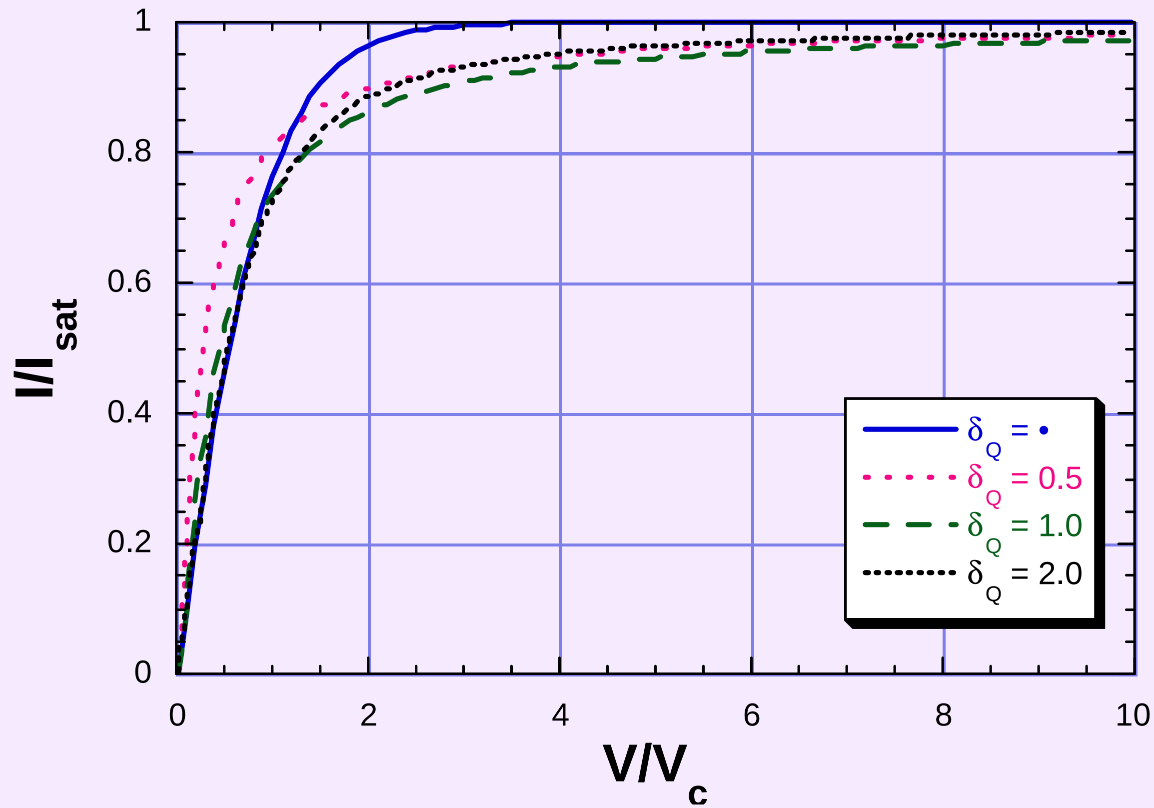


Electron and Hole Mobilities in Submicron CMOS

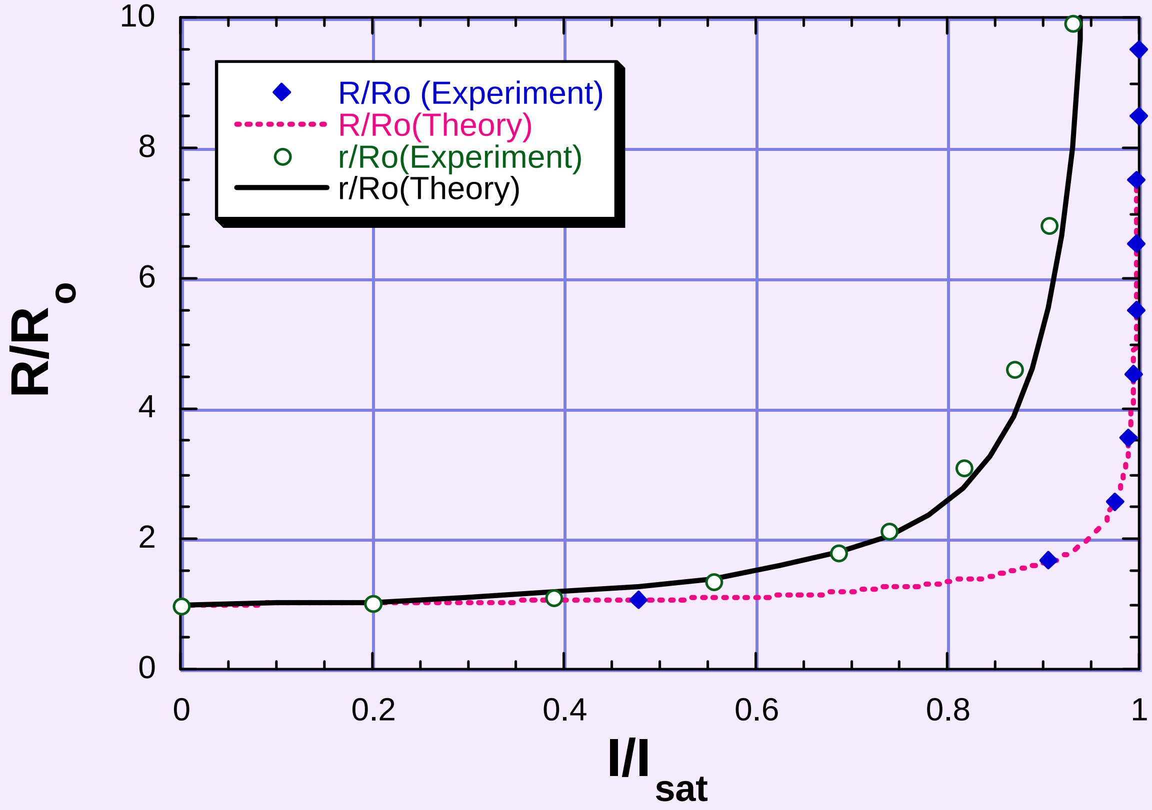


Courtesy: Y. Taur and E. Novak, IBM Microelectronics, IEDM97 Invited Talk.

Failure of Ohm’s Law



Resistance Blow-Up



20th Century CMOS Design

$$I_D = \mu_0 C_{Ox} \frac{W}{L}\left(V'_G - \tfrac{1}{2} V_{DS}\right)V_{DS}$$

$$\mu_{0n} C_{Ox} \frac{W_n}{L_n} = \mu_{0p} C_{Ox} \frac{W_p}{L_p}$$

$$L_p = L_n$$

$$W_p = \frac{\mu_{0n}}{\mu_{0p}} W_n \approx 2W_n$$

21st Century CMOS Design

$$I_D = \mu_0 C_{Ox} \frac{W}{L} \frac{1}{1 + {V_{DS}}/{V_C}} \left(V_G' - \tfrac{1}{2} V_{DS} \right) V_{DS}$$

$$\mu_{0n} C_{Ox} \frac{W_n}{L_n} = \mu_{0p} C_{Ox} \frac{W_p}{L_p}$$

$$V_{cn} = V_{cp} = L_n \frac{v_{sat}}{\mu_{0n}} = L_p \frac{v_{sat}}{\mu_{0p}}$$

$$L_n = L_p \frac{\mu_{0n}}{\mu_{0p}} \approx 2 L_p \qquad W_n = W_p$$

Conclusions

- Electric field puts an order into otherwise completely random motion
- An electron has apparent higher energy on a tilted band diagram
- Einstein ratio with degraded mobility results in apparent high electron temperature
- Electric field at the gate of a MOSFET does not heat electrons (quantum-confined mobility degradation)
- RC time constants will dominate over transit time delay because of enhanced resistance